Investigating the Relationship Between the Length of a Cantilevered Ruler and Its Fundamental Natural Frequency of Vibration

Javier Huang

Abstract

This research investigation examined the relationship between the length of a cantilevered ruler and its fundamental natural frequency of vibration. A cantilever beam model was used to describe the system, where bending-induced restoring forces produce oscillatory motion governed by beam stiffness and mass distribution. The theoretical framework predicts that the fundamental frequency scales inversely with the square of the beam length, $f \propto \frac{1}{l^2}$, based on standard beam vibration theory. The ruler was clamped at varying effective lengths between 3.40 cm and 8.40 cm in 0.20 cm increments, and vibrational motion was induced by a brief impulse. The resulting oscillations were recorded using a smartphone microphone and analyzed through audio frequency extraction methods to determine the dominant frequency component for each length. The dependence of frequency on length was then evaluated against the theoretical model. The results showed a clear decrease in fundamental frequency with increasing length, consistent with the predicted inverse-square relationship. Error analysis indicated an exponent deviation of approximately 7% ± 1% from the theoretical value, with uncertainties having minimal impact on overall trend consistency. While the model held across the measured range, extrapolation beyond the experimental domain showed divergence from ideal predictions, suggesting limitations in both measurement sensitivity and beam idealization assumptions. The findings support the theoretical relationship between cantilever length and natural frequency within experimental uncertainty.